CLAREMONT CENTER for MATHEMATICAL SCIENCES

CLAREMONT MATHEMATICS WEEKEND

Schedule of Events
Day 1 (January 30, 2016)

9:00 – 9:30 à Registration / Coffee and Snacks
9:30 – 9:45 à Welcome Speech (Alfonso Castro)
9:45 – 12:00 Presentations
12:00 – 1:30 Lunch Break
1:30 – 3:10 Presentations
3:10 – 3:40 Coffee Break
3:40 – 5:30 Presentations
5:20 – 6:00 Wine and Cheese

Day 2 (January 31, 2016)

9:00 – 9:30 à Coffee and Snacks
9:30 – 11:55 Presentations
12:00 – 12:45 Closing Remarks
12:45 – 2:00 Lunch Break
2:00 – 6:00 Math Competition

Speaker: Erica Flapan (Pomona College)

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Publications:

- Flapan, Erica, and Kenji Kozai. "Linking number and writhe in random linear embeddings of graphs." *Journal of Mathematical Chemistry* 54.5 (2016): 1117-1133.
- Flapan, Erica, and Gabriella Heller. "Topological complexity in protein structures." *Molecular Based Mathematical Biology* 3.1 (2015).

Title: Spatial Graph Theory

Abstract:

Spatial graph theory, which is the study of embeddings of graphs in R^3 and other 3-manifolds, grew out of the study of non-rigid molecules. However, because it considers embeddings of 1-dimensional objects in 3-dimensional manifolds, the field is also closely related to knot theory and low dimensional topology. However, in contrast with knots, the intrinsic structure of a graph can play a key role in determining the topo-logical properties of its embed-dings. For example, there are graphs which have the property that all of their embeddings in R^3 contain a non-trivial knot or link, and other graphs which have the property that no embedding of them in R^3 is invariant under an orientation reversing homeomorphism of R^3. This talk will present a survey of some of the open problems in spatial graph theory and its applications.

Speaker: Henry Schellhorn (CGU)

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Publications

• Schellhorn, Henry, and Ran Zhao. "A String Model of Liquidity in Financial Markets." *arXiv preprint arXiv:1608.05900* (2016).
• Jin, Sixian, Qidi Peng, and Henry Schellhorn. "A representation theorem for smooth Brownian martingales." *Stochastics* 88.5 (2016): 651-679.

Title: Semi-group Solution of Second Order Fully Nonlinear Parabolic Partial Differential Equations
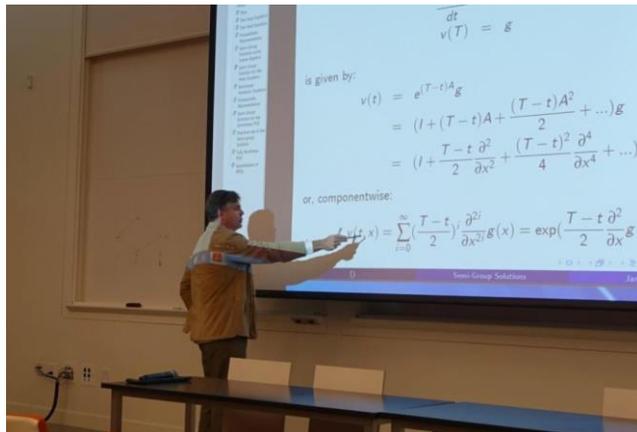
Abstract:

The Hamilton-Jacobi-Bellman equation (HJB) of stochastic control is an example of a fully nonlinear parabolic second-order PDE. As far as we know, a semi-group theory has not yet been proposed for this type of equation. One application of the semi-group theory is to give conditions for local existence of the PDE and, in some very particular cases, for global existence. The advantage of the semi-group method is that the existence proof is constructive, and thus leads automatically to Picard-type algorithms for the numerical computation of solutions. For the HJB equation, this method is similar to the policy iteration method, which was developed only in the discrete-time setting.

We obtain a semi-group solution by combining two new results from stochastic analysis. First, we use a result by Cheridito, Soner, Touzi, and Victoir (2006) which characterizes the solution of that PDE as a second-order backward stochastic differential (2BSDE). Since there is a martingale solution to the 2BSDE, the solution of the PDE is directly related to the martingale solving the corresponding BSDE. Secondly, we reuse one of our results (Jin, Peng, and Schellhorn 2015) , where we gave a new representation of a Brownian martingale as the exponential of a time-dependent generator, applied to the terminal value.

Speaker: Lenny Fukshansky

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Publications

- Böttcher, Albrecht, et al. "Toeplitz determinants with perturbations in the corners." *Journal of Functional Analysis* 268.1 (2015): 171-193.
- Fukshansky, Lenny, Stephan Ramon Garcia, and Xun Sun. "Permutation invariant lattices." *Discrete Mathematics* 338.8 (2015): 1536-1541.

# Title: Arithmetic Lattices and Elliptic Curves

Abstract:

We will review the classical construction of elliptic curves from lattices in the plane, focusing specifically on the important class of arithmetic lattices. To this end, we will discuss the connection between certain properties of elliptic curves and their corresponding lattices, as well as provide counting estimates for planar arithmetic lattices, up to a certain natural equivalence. Bridging arithmetic geometry and analytic number theory, this talk will feature group actions, fundamental domains, the modular j-invariant, and height functions. This is joint work with Pavel Guerzhoy (Hawaii) and Florian Luca (Wits, South Africa).

Speaker: Asuman Aksoy

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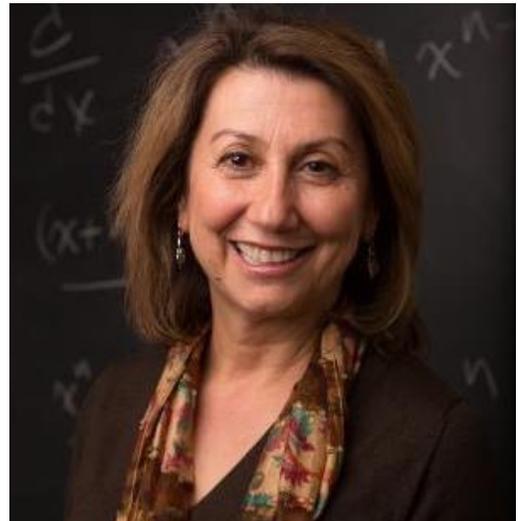
Publications
- Aksoy, Asuman Güven, and Jose M. Almira. "On Montel and Montel–Popoviciu theorems in several variables." *Aequationes mathematicae* 89.5 (2015): 1335-1357.
- Aksoy, Asuman Güven, and Grzegorz Lewicki. "Bernstein's Lethargy Theorem in Fréchet Spaces." *Journal of Approximation Theory* 209 (2016): 58-77.

Title: The Banach-Tarski Paradox

Abstract:

The Banach-Tarski paradox states that a ball in 3-dimensional space may be decomposed and then reassembled by rigid motions as two balls of the same size as the original. At first glance, the paradox may seem preposterous since it strongly contradicts our intuition about the conservation of mass and volume. In this talk, rather than prove the Banach-Tarski decomposition, we give Vitali's short proof of the existence of non-measurable sets. This helps us understand the paradox by showing that we cannot define volume for all subsets, but rather, only for some "good" subsets.

Speaker: Ivan Ventura

Department of Mathematics

Harvey Mudd College

Visiting Professor

Title: Can you "hear" the shape of a radial Schrödinger operator?

Abstract:

Inverse problems are a large class of both theoretical and applied problems that have captivated the mathematical community for over half a century. During this time numerous applications have arisen in in a variety of fields, such as medical imaging and cloaking. In the first half of this talk I will discuss, by example, general inverse problems and how they arise in the real world. In the second half I will focus specifically on spectral inverse problems, starting with the classic "Can you hear the shape of a drum?" problem in the case of the sphere. Finally I will present an analogous result for semiclassical Schr\"odinger operators

Speaker: Sam Nelson

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Publications

- Nelson, Sam, and Patricia Rivera. "Bikei invariants and gauss diagrams for virtual knotted surfaces." *Journal of Knot Theory and Its Ramifications* 25.03 (2016): 1640008.
- Nelson, Sam, and Jake Rosenfield. "Bikei homology." *arXiv preprint arXiv:1603.02719* (2016).
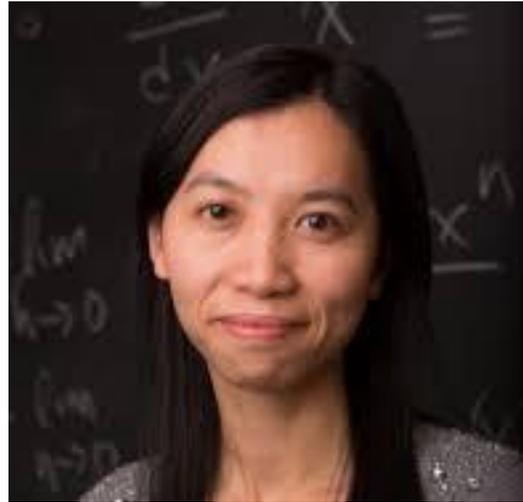
Title: Quandles and Knots
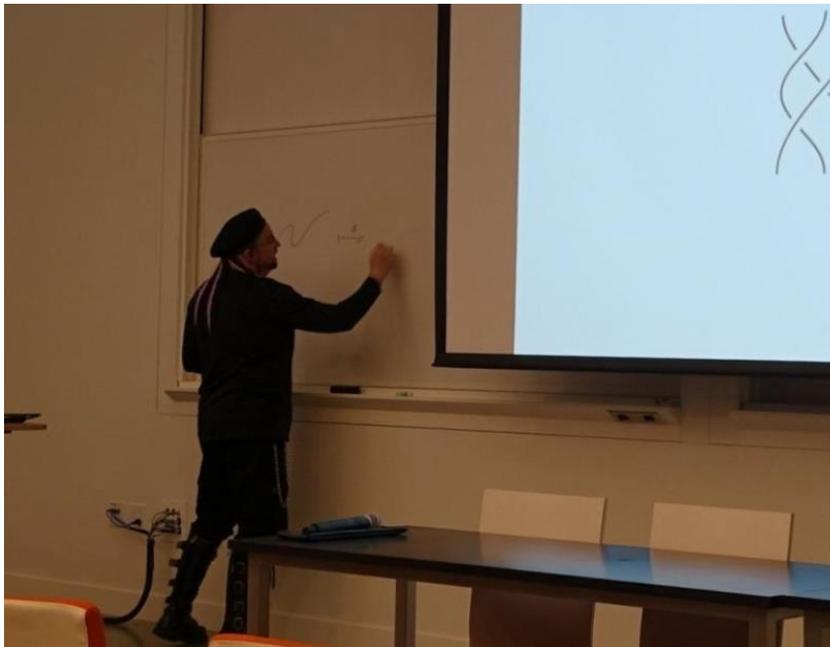
Abstract:

Quandles are algebraic structures related to knots. In this talk we will see the basics of quandle theory and its applications to knot theory

Speaker: Chiu-Yen Kao

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Publications

• Computational Methods for Extremal Steklov Problems by Eldar Akhmetgaliyev, Chiu-Yen Kao, and Braxton Osting, SIAM Journal on Control and Optimization, 55(2), pages 1226-1240, 2017
• CY Kao, CW Shih, CH Wu . "Absolute stability and synchronization in neural field models with transmission delays." Physica D: Nonlinear Phenomena 328, 21-33

Title: Computational Methods for Extremal Steklov Problems

Abstract:

We develop a computational method for extremal Steklov eigenvalue problems and apply it to study the problem of maximizing the p-th Steklov eigenvalue as a function of the domain with a volume constraint. In contrast to the optimal domains for several other extremal Dirichlet- and Neumann-Laplacian eigenvalue problems, computational results suggest that the optimal domains for this problem are very structured. We reach the conjecture that the domain maximizing the p-th Steklov eigenvalue is unique (up to dilations and rigid transformations), has p-fold symmetry, and an axis of symmetry. The p-th Steklov eigenvalue has multiplicity 2 if p is even and multiplicity 3 if p>=3 is odd.

Speaker: Deanna Needell

(Claremont McKenna College)

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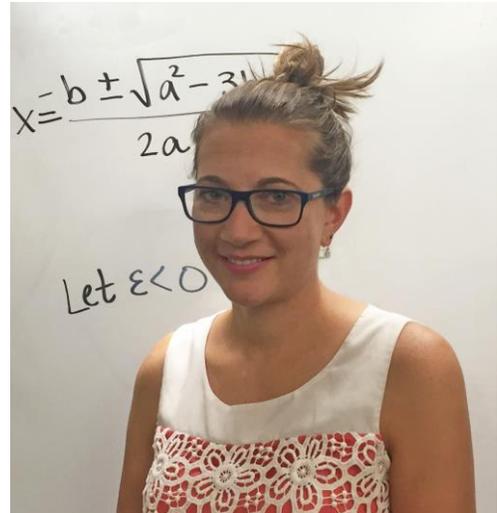
Recent Publications

- Giryes, Raja, and Deanna Needell. "Greedy signal space methods for incoherence and beyond." *Applied and Computational Harmonic Analysis* 39.1 (2015): 1-20.
- Davenport, Mark A., et al. "Constrained adaptive sensing." *IEEE Transactions on Signal Processing* 64.20 (2016): 5437-5449.

Title: Why it's hot in high dimensions

Abstract:

We live in a three- dimensional world, so the geometry in low dimensional space is relatively intuitive to us. In this talk we will explore those intuitions as we progress into higher and higher dimensions, where we will find beautiful and surprising geometries. We will discuss how these geometric notions can be helpful in real- world applications like the big data crisis. In the end, students will have an appreciation for the beauty and challenges of high dimensional mathematical ideas as well as how those appear in their day to day lives.

Speaker: Qidi Peng

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Recent Publications

- Jin, Sixian, Qidi Peng, and Henry Schellhorn. "A representation theorem for smooth Brownian martingales." *Stochastics* 88.5 (2016): 651-679.
- Aksoy, Asuman G., and Qidi Peng. "On a theorem of SN Bernstein for Banach spaces." *arXiv preprint arXiv:1605.04592* (2016).

Title: The Beauty of Fractal Geometry: Self-Similarity

Abstract:

I would reveal a totally new world of plastic beauty to you. This talk introduces basic concepts in fractal geometry: Hausdorff dimension, fractal, self-similar process. And it promotes a new mathematical and philosophical synthesis. Applications to financial modeling are given to show one face of interests of this field.

Speaker: Christopher Towse

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Recent Publications
- Weierstrass Points on Cyclic Covers of the Projective Line, Trans. Amer. Math. Soc. 348 (1996) 3355–3378.
- Generalized Wronskians and Weierstrass Weights, Pac. J. Math., 193, no.2 (2000), 501–508.

Title: Some Basics of Analytic Number Theory:
Functions that encode Arithmetic Data

Abstract:

Many fundamental properties of integers can be described in terms of arithmetic functions. (Is a number a square? Is a number a prime? How many divisors does a number have?) Arithmetic functions can be used as a way to describe many fundamental properties of integers: Is a number a square? Is a number a prime? How many divisors does a number have? We can encode this information using types of generating functions. In this talk we will see how to encode and decode number theoretic information using Dirichlet series and their Euler products. We will look at the resulting functions, including the famous Riemann zeta function, which has a million dollar prize attached to it.

Speaker: Marina Chugunova

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Publications

- M. Chugunova, H. Volkmer, Spectral analysis of an operator arising in fluid dynamics, Studies in Applied Mathematics, 123 (3), 291 - 309, (2009)
- T. Azizov, M. Chugunova, Pontryagin theorem and analysis of the spectral stability of solitons, Mat.Zametki, 8 (5), 643 - 658, (2009)

Title: Two-Pulse Solutions in the Fifth-Order KdV Equation
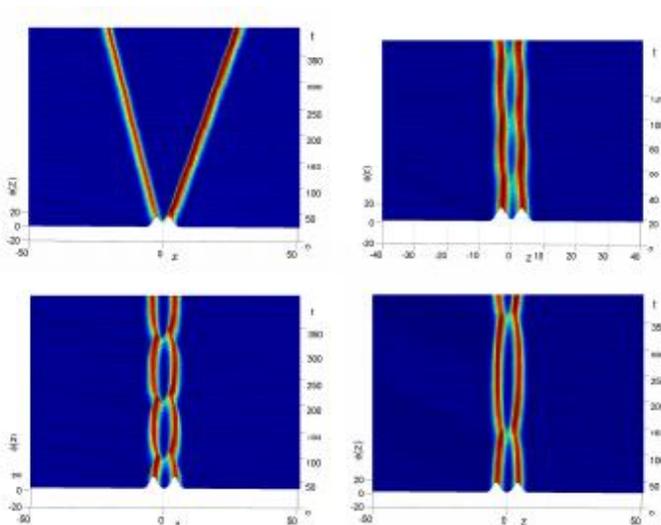
Abstract:

Using Pontryagin space structure we show spectral stability of the two-pulse solutions for fifth-order KdV. Eigenvalues of the linearized problem are approximated numerically in exponentially weighted spaces where embedded eigenvalues are isolated from the continuous spectrum. Approximations of eigenvalues and full numerical simulations of the fifth-order KdV equation confirm stability of two-pulse solutions associated with the minima of the effective interaction potential and instability of two-pulse solutions associated with the maxima points.

Speaker: Mark Huber

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Publications

- M. Huber, "The Fundamental Theorem of Perfect Simulation", arXiv:1704.03561(2017)
- M. Huber, "An estimator for Poisson means whose relative error distribution is known",arXiv:1605.09445(2016)

Title: A New Way of Estimating the Probability of Heads on a Coin

Abstract:

Given a coin with unknown probability of heads and the ability to flip the coin as often as one wishes, what is the best way of estimating the chance that a coin flip is heads? In this talk I will present a new estimator for this problem with the important property that the relative error of the estimate does not depend on the quantity being estimated.

Speaker: Alfonso Castro

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Publications

- "Nonexistence of Continuous Solutions for a Semilinear Wave Equations with Smooth Data, Water Waves, ed. M. F. Mahmood, D. Henderson and H. Segur, World Scientific (2010).
- (With C. Chang) "A Variational Characterization of the Fucik Spectrum and Applications, Revista Colombiana de Matemáticas, Vol. 44, No. 1, (2010), pp. 23-40.

Title: Infinitely Many Solutions to a Semilinear Laplace-Beltrami Equation

Abstract:

We prove the existence of infinitely many rotationally symmetric solutions to a semilinear Laplace-Beltrami equation on spheres. The techniques are one-dimensional arguments that include phase plane analysis and a Pohozaev identity. The talk is based on joint paper with Emily Fischer (HMC-14) published in the Canadian Mathematics Bulletin (2015).

Speaker: Weiging Gu

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Publications

- de Pillis, Lisette G., Weiqing Gu, and Ami E. Radunskaya. "Mixed immunotherapy and chemotherapy of tumors: modeling, applications and biological interpretations." *Journal of theoretical biology* 238.4 (2006): 841-862.
- de Pillis, Lisette G., et al. "Chemotherapy for tumors: An analysis of the dynamics and a study of quadratic and linear optimal controls." *Mathematical Biosciences* 209.1 (2007): 292-315.

Title: Differential Geometric Data to Decision

Abstract:

There are big data problems everywhere in this world. There is an urgent need to apply new and advanced mathematical techniques to extract knowledge and insights from large and complex collections of digital data since traditional statistical methods do not suffice. In this talk, I will present how to use the techniques in differential geometry especially using manifold and Lie group theories for approaching big data problem and for data-to-information and data-to-decision. I will use two to three examples that I worked with three of my Claremont students to demonstrate how to use differential geometric techniques to visualize data, reduce data dimension, extract data features, and define appropriate distance functions to analyze time series data. You will see Grassmann manifolds, Stiefel manifolds, and the Lie group of rigid motions play important roles in processing and analyzing big nonlinear dynamical data.