CLAREMONT CENTER
for MATHEMATICAL SCIENCES

POSTER SESSION FALL 2019

Title: The Kohn-Laplacian and Complex Green Operator on Spheres
Authors: Elena Kim*, Jacob Ogden, Tommie Reerink, Yunus Zeytuncu
Affiliation(*): Pomona College
Abstract: The Kohn Laplacian ($square_b$) is an important second order differential operator defined on a CR manifold, and it is closely related to the geometry of the underlying manifold. We study a simpler operator, $\mathcal{Q}$, on S^7 and find a relationship between its eigenvalues and those of $square_b$; Expanding on $\mathcal{Q}$, we examine a new operator $\mathcal{R}$ and are able conclude that on $\mathbb{S}^{2n-1}$, $\mathcal{R}$ coincides with $square_b$, thereby deriving the spherical case of the Folland-Stein-Tanaka formula for $square_b$ on strongly pseudoconvex CR manifolds. Continuing our study of $\mathbb{S}^7$, we take inspiration from Rossi's nonembeddable perturbation of the CR structure on $\mathbb{S}^3$ to construct a family of abstract CR structures on $\mathbb{S}^7$ that are not of hypersurface type which we conjecture are no longer embeddable into $\mathbb{C}^n$ for any n. The complex Green operator, $\mathcal{G}$, is the inverse to $square_b$ modulo its kernel. We prove Schatten estimates for $\mathcal{G}$, showing that on $\mathbb{S}^{2n-1}$, $\mathcal{G}$ has finite Schatten r-norm if and only if $r > n$. We then obtain a new proof of the Sobolev estimate

$$\|\mathcal{G}u\|_{s+1}^2 \leq c\|u\|_s^2$$

for $s \geq 0$ and show that this is the best possible estimate.

Title: Optimization of Inhomogeneous Buckled Rods and Plates
Authors: Alexa Bayangos*, Chiu-Yen Kao
Affiliation(*): Pomona College
Abstract: Optimizing eigenvalues of biharmonic equations has many applications including frequency control of rods and plates based on density distribution and maximizing the critical buckling load. This research aims to find the optimal cross-sectional area of an elastic column with given length and volume to maximize the critical buckling load. By using finite difference methods, we solved the forward problem on domains in one and two dimensions. We then evaluated the accuracy and robustness of our numerical approach by comparing our results to previous analytical studies. Lastly, we developed a numerical algorithm to identify the optimal cross-sectional area function which maximizes the k-th eigenvalue on any given general domain.

Title: Arbitrary Rectangle-range Generalized Elastic Net Penalty Model and Variable Selection Consistency
Authors: Yujia Ding*, Zhengming Song, Qidi Peng, Hansen Chen
Affiliation(*): Claremont Graduate University
Abstract: This paper proposes an arbitrary rectangle-range generalized elastic net penalty least squares method for variable selection in high dimensional sparse linear regression models. This method is an extension of the nonnegative elastic net penalty method and is shown to have variable selection consistency and estimation consistency under certain condition which extends the Nonnegative Irrepresentable Condition in elastic net. Note that to get the solution of the arbitrary rectangle-range generalized elastic net model, many algorithms such as Lars and coordinate decent can be used, among which multiplicative updates approach is preferred since it is faster and simpler. The constrained index tracking problem in stock market without short sales is studied in the latter part. The tracking results indicate that arbitrary rectangle-range generalized elastic net model can get small tracking error and is successful in assets selection.

Title: Primitive Root Bias for Twin Primes: Totient Quotients and the Sum-of-Divisors Function
Authors: Gabe Udell*, Stephan Ramon Garcia, Florian Luca, Kye Shi
Affiliation(*): Pomona College
Abstract: Garcia, Kahoro, and Luca showed that the Bateman–Horn conjecture implies $\varphi(p-1) \geq \varphi(p+1)$ for a majority of twin-primes pairs $p, p+2$ and that the reverse inequality holds for a small positive proportion of the twin primes. That is, p tends to have more primitive roots than does $p+2$. We prove that Dickson's conjecture, which is much weaker than Bateman–Horn, implies that the quotients $\frac{\varphi(p+1)}{\varphi(p-1)}$, as $p, p+2$ range over the twin primes, are dense in the positive reals. We also establish several Schinzel-type theorems, some of them unconditional, about the behavior of $\frac{\varphi(p+1)}{\varphi(p)}$ and $\frac{\sigma(p+1)}{\sigma(p)}$, in which σ denotes the sum-of-divisors function.

Title: An Analogue of K-marked Durfee Symbols for Strongly Unimodal Sequences
Authors: Savana Ammons*, Laura Seaberg, Young Jin Kim
Affiliation(*): Harvey Mudd College
Abstract: This paper applies methods from Andrews's work on partitions to another combinatorial object: strongly unimodal sequences. Specifically, we define "k-marked unimodal symbols" for unimodal sequences analogously to how Andrews defines k-marked Durfee symbols for partitions. We establish a multivariate rank generating function $U_k(\zeta_{\mathbf{k}}; \mathbf{q})$ for k-marked unimodal symbols, as well as $\mathcal{SCU}_k(q)$ for self-conjugate k-marked unimodal symbols, which we also interpret combinatorially in terms of partitions. We then discuss potential quantum modularity properties for $U_k(\zeta_{\mathbf{k}}; \mathbf{q})$ for certain vectors of roots of unity $\zeta_{\mathbf{k}}$, including determining when $U_k(\zeta_{\mathbf{k}}; \mathbf{q})$ can be defined as a function on a subset of rationals. We conclude with some further observations based on computational data and a congruence conjecture about the full rank.

Title: Modeling Type I diabetes: Parametric sensitivity data driven approach with multi-compartment
Authors: An Do*, Amber Nguyen, Nathaniel Efrat-Henrici, Matt Matuslewicz, Lisette de Pillis, Blerta Shtylla
Affiliation(*): Claremont Graduate University
Abstract: It has been documented in recent studies (Anderson 2013, Shtylla 2019) that an appropriate injection of a dendritic cell (DC) vaccine could halt or slow the progression of autoimmune diabetes in mice. Previous mathematical models involving ordinary differential equations (ODEs) have previously done parametric analysis on a single compartment of the pancreas to model the development of simulated diabetogenesis. This study integrates two new compartments (the spleen and the bloodstream) to develop a much more complex and accurate model with the foundation of a previously developed single compartment model. With proper analysis, parameter sensitivity on this mathematical model is a conduit for determining effective potential treatments.

Title: How to Write a Textbook: Representation Theory for Undergraduates
Authors: Bella Senturia*, Gizem Karaali
Affiliation(*): Pomona College
Abstract: Professor Karaali is writing an undergraduate Representation Theory textbook with only linear and abstract algebra prerequisites. The goal is that this book be accessible to students in their first years of mathematical training. I served as a student consultant in the textbook development process, combing through the current draft and giving Professor Karaali opinions and feedback on things as specific as her word choice and as general as thoughts about the structure of chapters or her style of mathematical writing. My previous representation theory course enabled me to give opinions as a student with previous material exposure for the first, more completed half of her manuscript. The second half was mostly concepts I had not seen before and I provided the perspective of a student seeing the material for the first time, trying to learn it directly from the book without assistance of a class curriculum or instruction. My previous math courses gave me extensive exposure to various math textbook types, which coupled with my efficacy in conveying complex ideas, helped Dr. Karaali work towards book completion.

Title: Inverse Semigroups Associated with Markov Subshifts
Authors: Aria Beaupre*, Anthony Dickson, David Milan, Christin Sum
Affiliation(*): Claremont McKenna College
Abstract: We answered the question "Under what conditions is an inverse semigroup isomorphic to the inverse hull of a Markov sub-shift?". We then explored the implications of this to better understand the relationship between two Markov sub-shifts with isomorphic inverse hulls.

Title: On (t, r) Broadcast Domination of Certain Grid Graphs
Authors: Natasha Crepeau*, Zetta
Affiliation(*): Harvey Mudd College
Abstract: If $G = \langle V(G), E(G) \rangle$ is a connected graph, then a set $D \subset V(G)$ is dominating if every vertex of $V - D$ has at least one neighbor in D. A generalization of this concept is (t, r) broadcast domination. In this setting certain vertices are designated as towers of signal strength t, which send out signal to neighboring vertices decaying linearly as the signal traverses the edges of the graph. We let $\mathbb{T}$ be the set of all towers and we define the signal received by a vertex $v \in V(G)$ as $f(v) = \sum_{w \in \mathbb{T}} max(0, t - d(v, w))$, where w is a tower and $d(v, w)$ denotes the distance between v and w. Blessing, Insko, Johnson, Mauretour (2014) defined a (t, r) *broadcast dominating set*, or a (t, r) *broadcast*, on G as a set $\mathbb{T} \subseteq V(G)$ such that $f(v) \geq r$ for all $v \in V(G)$. The minimal cardinality of a (t, r) broadcast on G is called the (t, r) **broadcast domination number** of G. In this poster, we present our research on the (t, r) broadcast domination number for graphs including paths, grid graphs, and different lattices.

Title: Prevalence and Propagation of Fake News
Authors: Bhavana Bheem*, Daniela Elizondo, Deyana Marsh, Steven Witkin
Affiliation(*): Harvey Mudd College
Abstract: Social media is unique in that its users play a decisive role in the content that gets propagated to other users. In recent years, a tremendous amount of research has gone into understanding how unreliable, or 'fake news', propagates through social media. Scholars have raised concerns on the effects that fake news has on our political sphere, and our democracy as a whole. Our research project is interested in characterizing how news propagates through social media so that we can better understand the intentions of a malicious agent who wishes to spread fake news. We found that there is a lot of variability in what content gets propagated, but that more truthful news tends to be propagated more than less truthful news.

Title: Topography and Behavior Based Movement Modeling for Missing Persons in Land-Wilderness Settings
Authors: Madeline Brown*, J. Alanis, J. Kitchens, J. Magaa, C. Velastegui, M. Thakura, L. Arriola, B. Espinoza, A. Murillo, M. Rodriguez-Messan, R. Koester, and C. Castillo-Garsow
Affiliation(*): Scripps College
Abstract: Search and Rescue (SAR) operations are critical to the safety and well-being of individuals who visit state and national parks. In a missing hiker case, time is crucial as survival rates decrease each hour. It is estimated that approximately 4080 individuals become lost every year in the US. The average cost of a SAR operation is $1,375 per person, and the cost of each mission is increasing yearly. Most SAR operations are based on rings of probabilistic distances. There is a need to incorporate a mechanistic, mathematical model that takes human behavior into consideration for determining SAR operations. Data from resources such as the International Search and Rescue Incident Database are analyzed to identify patterns in human behaviors and key geographic influences to develop a mechanistic model of missing persons. We use a discrete-time Markov Decision Process, where the lost individuals state is used to determine a personal strategy for being found. The individual then interacts with the environment, where a utility function for that strategy over the geographic environment determines direction of travel. We take incident reports in various national parks as a case study to test our model. Implications are discussed for SAR, hiker survival training, and other aspects. The proposed model may be extended to the prediction of path-tracing of specific groups of people including experienced hikers or individuals who suffer from mental illnesses.

Title: Cluster Analysis on Locally Asymptotically Self-similar Processes with Known Number of Clusters
Authors: Ran Zhao*, Qidi Peng, Nan Rao
Affiliation(*): Claremont Graduate University
Abstract: We study the problems of clustering locally asymptotically self-similar stochastic processes, when the true number of clusters is priorly known. A new covariance-based dissimilarity measure is introduced, from which the so-called approximately asymptotically consistent clustering algorithms are obtained. In a simulation study, clustering data sampled from multifractional Brownian motions is performed to illustrate the approximated asymptotic consistency of the proposed algorithms.

Title: Inversion of Laplace transform of a linear combination of point masses
Authors: Adam Guo*, Hrushikesh Mhaskar
Affiliation(*): Pomona College
Abstract: The objective of this research project is to investigate a new approach for the inverse problem of finding a finitely supported measure on the real line given samples of its Laplace transform. The problem appears, for example, in the analysis of magnetic resonance relaxometry data. This is a notoriously ill-posed problem. Existing algorithms in the literature require a prior knowledge of the number of points in the support of the measure. We propose an alternative method to determine this number as well as to determine the measure itself. We used the samples of the Laplace transform at the zeros of the so-called Hermite polynomial of sufficiently large degree. Chui and Mhaskar have given an algorithm that can be used to determine in one stroke both the number of points in the support of the measure as well as the locations of points in this support. A comparison of the height of the peaks with the on-diagonal values of a reference spectrum yields the weights that the measure attaches to each of the points in its support.

Title: Knot Invariants & Kaestner Brackets
Authors: Forest Kobayashi*
Affiliation(*): Harvey Mudd College
Abstract: Given two tame knots K_1, K_2, we can prove $K_1 \cong K_2$ by exhibiting a finite sequence of Reidemeister moves taking K_1 to K_2. However, in general, proving $K_1 \ncong K_2$ is not so straightforward. To address this problem we employ *knot invariants*, i.e. systematic ways of assigning "nice" values to knots such that equivalent knots get mapped to the same thing. Note, by contrapositive, if two knots are assigned *different* values then they can(k)not be equivalent. Thus, invariants offer us a coarse method for distinguishing knots. In this poster, I will give a high-level introduction to the importance of knot invariants, and offer a brief summary of two particularly fruitful families of invariants (birack-flavored and skein-based). I will also summarize the important results from my summer research (which examines methods for combining these two families and manipulating them computationally), and conclude by discussing some directions for further research.

Title: Traveling waves in thin film equations with source
Authors: Yadong Ruan*, Ali Nadim, Marina Chugunova
Affiliation(*): Claremont Graduate University
Abstract: We examine the dynamics of a thin film formed by a distributed liquid source on a vertical solid wall. The model is derived using the lubrication approximation and includes the effects of gravity, upward airflow and surface tension. When surface tension is neglected, a critical source strength is found below which the film flows entirely upward due to the airflow, and above which some of the flow is carried downward by gravity. In both cases, a steady state is established over the region where the finite source is located. Shock waves that propagate in both directions away from the source region are analyzed. Numerical simulations are included to validate the analytical results. For models including surface tension, numerical simulations are carried out. The presence of surface tension, even when small, causes a dramatic change in the film profiles and the speed and structure of the shock waves. These are studied in more detail by examining the traveling wave solutions away from the source region.

Title: Epistemic Logic of Know-Who
Authors: Sophia Epstein*, Pavel Naumov
Affiliation(*): Claremont McKenna College
Abstract: The paper proposes a trimodal logic that describes the interaction between three modalities: "knows", "for all agents", and "knows who". The key results are the soundness and the completeness theorems for this logical system with respect to a proposed set of semantics.

Title: Frobenius Problem in Totally Real Number Fields: An Exploration of Geometry and Number Theory
Authors: Yingqi (Edward) Shi* and Lenny Fukshansky
Affiliation(*): Claremont McKenna College
Abstract: Frobenius problem and its many generalizations have been extensively studied in several areas of mathematics. We study semigroups of totally positive algebraic integers in totally real number fields, defining analogues of the Frobenius numbers in this context. We use a geometric framework recently introduced by Aliev, De Loera and Louveaux to produce upper bounds on these Frobenius numbers in terms of a certain height function. We investigate the smallest representation for members in the semigroups and we also use a result of Borosh and Treybig to obtain bounds on the size of representations and number of elements of bounded height in such positive semigroups of totally real algebraic integers.