Identification of the order of semilinear subdiffusion with memory

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In the last two decades, fractional partial differential equations play a key role in the description of the so-called anomalous phenomena. The signature of an anomalous diffusion is that the mean square displacement of the diffusing species $\langle(\mathbf{\Delta x})^2\rangle$ scales as a nonlinear power law in time, i.e. $\langle(\mathbf{\Delta x})^2\rangle \sim t^\nu$, $\nu > 0$. For a subdiffusive process, the value of ν is such that $0 < \nu < 1$, while for normal diffusion $\nu = 1$, and for a superdiffusive process, we have $\nu > 1$.

However, sometimes a value of the subdiffusion order is not given a priori. Here we discuss an approach to the reconstruction of a subdiffusion order ν from the state measurements for small time. To this end, for $\nu \in (0,1)$, we analyze initial boundary value problems for semilinear integro-differential equations on the multidimensional space domain $\Omega \subset R^n$ with the unknown $v = v(x,t)$:

$$\mathbf{D}_t^\nu v - \mathcal{L}_1 v - \int_0^t \mathcal{K}(t-s)\mathcal{L}_2 v(\cdot,s)ds = f(x,t,v) + g(x,t),$$

where $\mathbf{D}_t^\nu$ is the Caputo fractional derivative and $\mathcal{L}_1$, $\mathcal{L}_2$ are certain uniform elliptic operators of the second order with time-dependent smooth coefficients.

We provide two explicit formula reconstructing the order of the fractional derivative ν for small time state measurements. These formulas give rise to a regularization algorithm for calculating ν from possibly noisy measurements. We present several numerical tests illustrating the algorithm when it is equipped with quasi-optimality criteria for choosing the regularization parameters.

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