We study existence of nontrivial solutions of the elliptic system

$$\begin{cases} -\Delta u = au + bv + \dfrac{2\alpha}{\alpha+\beta} u|u|^{\alpha-2}|v|^{\beta}, & \text{in } \Omega; \\ -\Delta v = bu + cv + \dfrac{2\beta}{\alpha+\beta} |u|^{\alpha} v|v|^{\beta-2}, & \text{in } \Omega; \\ \quad u = v = 0, & \text{on } \partial\Omega, \end{cases} \tag{1}$$

for $\alpha > 1$, $\beta > 1$, $\alpha + \beta \leqslant 2^* = 2N/(N-2), N \geqslant 3$, $a, b, c \in \mathbb{R}$, and Ω an open bounded domain of $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$. We extend the work of Alves et al. (Nonlinear Analysis 42 (2000) 771-787), in the subcritical growth case: $\alpha+\beta < 2^*$, by proving that the boundary value problem (1) has at least two nontrivial solutions for the case in which the eigenvalues of the matrix $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ are higher than the first eigenvalue of the Laplacian over Ω with zero Dirichlet boundary conditions on $\partial\Omega$. We use variational methods and infinite-dimensional Morse theory to obtain the multiplicity result.